

Modal Hyperdoctrines

What i did in München



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LMU München

- A set of rules and a syntax
- Defines
 - Propositions
 - Context
 - Judgment

- Propositions are variables with arrows between them ($P \Rightarrow Q$)
- Contexts are lists of propositions ($\Gamma := P, (P \Rightarrow Q) \Rightarrow P$)
- Judgments are a list of propositions entails a proposition
 $(Q, P \Rightarrow Q \vdash P)$
- Rules like modus ponens

$$\frac{\Gamma \vdash P \Rightarrow Q \quad \Gamma \vdash P}{\Gamma \vdash Q}$$

- Propositional (Letters with arrows)
- Intuitionistic (We add OR, FALSE, etc)
- Classical (we add TND in the rules)
- First order (Terms, \forall and \exists)
- Linear logic
- *Guarded logic*
- Modal logics
- Anything you want

- We add a \triangleright «later» operator on propositions
- We add a next term which proves $A \Rightarrow \triangleright A$
- We add a fixpoint term which proves $(\triangleright A \Rightarrow A) \Rightarrow A$

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I was supposed to study that

«What is the mathematical structure behind a logic»

Logic	Models
Propositional	ST λ C / CCC
Classical propositional	Boolean algebras
Intuitionistic	Heyting algebras
First order	Hyperdoctrines

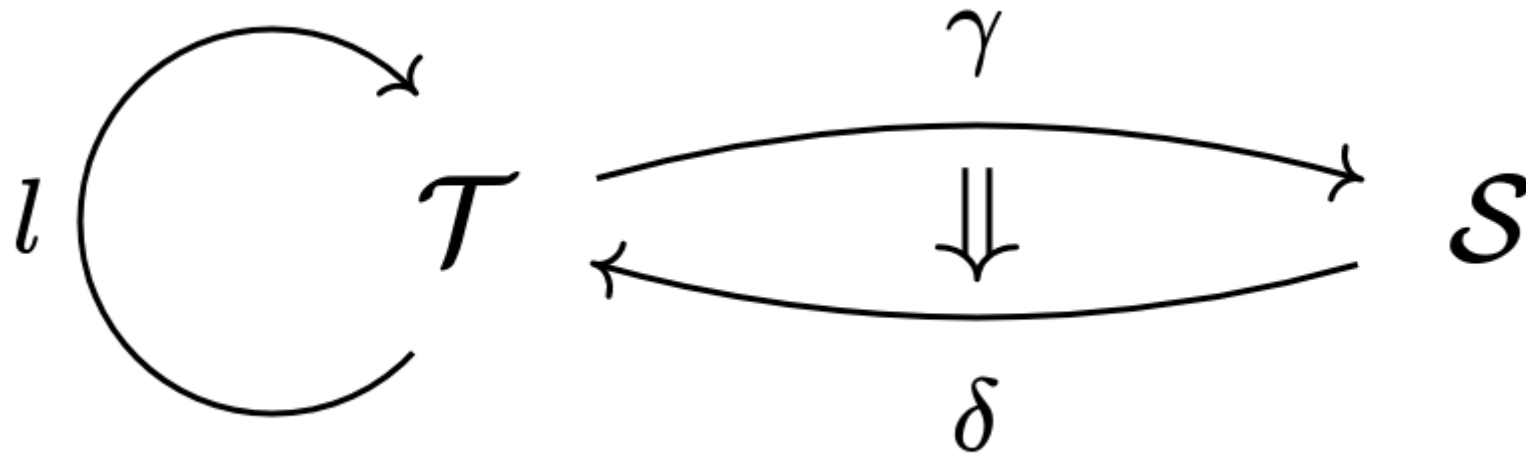
«What is the mathematical structure behind a logic»

Logic	Models
Propositional	ST λ C / CCC
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First order	Hyperdoctrines
Guarded Logic	?
Modal logics	?

- One model for Guarded logic
- Formalization in Lean by Møgelberg & Bahr
- Studied their formalization, copied it
- Is it really a model ?

«What is a model of Guarded Logic»

«What is a model of Guarded Logic»



- ✓ Implement Hyperdoctrines in Lean
- Implement Hyperdoctrines in Mathlib
- ✓ Show that ToT is a model of first order logic
- ✓ State the property of being a model of guarded logic
- Show that ToT is a model of guarded logic