Compilation and Program Analysis (#4) : Types, and Typing MiniWhile

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Lyon 1

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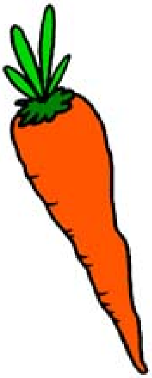
Typing



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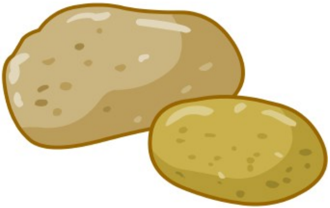
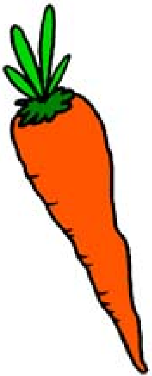


Typing



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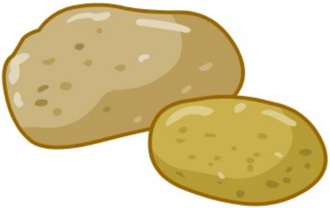


Typing

If you write: `"5432" + 1`

what do you want to obtain

- a compilation error? (OCaml)
- an exec error? (Python)
- the int 5433? (Visual Basic, PHP)
- the string `"54321"`? (Java)
- (too insane to put on a slide) (C, C++)
- anything else?

and what about `5432 / "1"` ?

Typing

When is

```
e1 + e2
```

legal, and what is its semantics?

- Typing: an analysis that gives a type to each subexpression, and reject incoherent programs.

When

- Dynamic typing (during execution): Lisp, PHP, Python
- Static typing (at compile time, after lexing+parsing): C, Java, OCaml

▶ Here: the second one.

Slogan

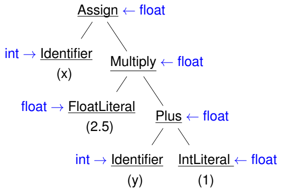
well typed programs do not go wrong

Typing objectives

- Should be **decidable**.
- It should reject programs like `(1 2)` in OCaml, or `1+"toto"` in C before an actual error in the evaluation of the expression: this is **safety**.

  The type system is related to the kind of error to be detected: **operations on basic types** / method invocation (message not understood) / correct synchronisation (e.g. session types) in concurrent programs / ...
- The type system should be expressive enough and not reject too many programs. (**expressivity**)

Principle



All sub-expressions of the program must be given a type

What does the programmer write?

- The type of all sub-expressions (like above) easy to verify, but tedious for the programmer
- Annotate only variable declarations (Pascal, C, Java, . . .)

  ```
  {int x, y; x = 2.5 * (y + 1);}
  ```
- Only annotate function parameters (Scala)

  ```
  def toto(y : Int) { var x = 2.5 * (y + 1) }
  ```
- Annotate nothing: complete inference : Ocaml, Haskell, . . .

  ```
  # let foo y = 2 * (y + 1);;
  val foo : int -> int = <fun>
  ```

Properties

- correction: “yes” implies the program is well typed.
- completeness: the converse.

(optional)

- principality : The most general type is computed.

Typing judgement

We will define how to compute **typing judgements** denoted by:

$$\Gamma \vdash e : \tau$$

and means "in environment Γ, expression e has type τ"

- Γ associates a type $\Gamma(x)$ to all declared variable x (that may or may not appear in e).

```
{
    int x = 42, y;
    float z;
}
```

$\leadsto$

$$\left\{ \begin{array}{lcl} x & \rightarrow & int, \\ y & \rightarrow & int, \\ z & \rightarrow & float \end{array} \right\}$$

Safety = well typed programs do not go wrong

In general a type-safety property looks like this:

Theorem (Safety)

If $\emptyset \vdash e : \tau$, then the reduction of e is infinite or terminates with a value.

Typing Safety

In general, a type-safety proof is based on two lemmas:

Lemme (progression)

If $\emptyset \vdash e : \tau$*, then* e *is a value or there exists* e' *such that* $e \rightarrow e'$*.*

Lemme (preservation)

If $\emptyset \vdash e : \tau$ *and* $e \rightarrow e'$ *then* $\emptyset \vdash e' : \tau$*.*

This works almost the same for small-step and big-step.

2 Imperative languages (C, Mini-While)

- Simple Type Checking for mini-while
- More advanced typing features

Mini-While Syntax

Expressions (with boolean expression to make typing interesting):

$$
\begin{array}{rcll}
e & ::= & c & \textit{constant} \\
 & | & x & \textit{variable} \\
 & | & e + e & \textit{addition} \\
 & | & e \times e & \textit{multiplication} \\
 & | & e < e & \textit{boolean expression} \\
 & | & \ldots &
\end{array}
$$

Mini-while:

$$
\begin{array}{rcll}
S(Smt) & ::= & x := expr & \text{assign} \\
 & | & skip & \text{do nothing} \\
 & | & S_1; S_2 & \text{sequence} \\
 & | & \texttt{if } e \texttt{ then } S_1 \texttt{ else } S_2 & \text{test} \\
 & | & \texttt{while } e \texttt{ do } S \texttt{ done} & \text{loop}
\end{array}
$$

Typing rules for expr

Here types are basic types: Int|Bool

$$\frac{}{\Gamma \vdash x : \Gamma(x)} \qquad \frac{n \in \mathbb{Z}}{\Gamma \vdash n : \texttt{int}}$$ (or $\mathrm{tt}\text{: bool}, \dots$)

$$\frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 + e_2 : \texttt{int}} \qquad \frac{\Gamma \vdash e_1 : \texttt{int} \qquad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 < e_2 : \texttt{bool}}$$

$$\frac{\Gamma \vdash e_1 : \texttt{bool} \qquad \Gamma \vdash e_2 : \texttt{bool}}{\Gamma \vdash e_1 \wedge e_2 : \texttt{bool}} \qquad \dots$$

Example

show that $(x + 42)$ **is well typed in an environment where** $[x \mapsto int]$

Typing rules for statements: $\Gamma \vdash S$

A statement S is well-typed (there is no type for statements)

on board!

Typing: an example

Considering $\Gamma = [x \mapsto int]$, prove that the given sequence of instructions is well typed:

```
x = 3 ;
x = x+9 ;
```

on board!

Problem: how to define Γ in mini-while? (1/2)

Possible solution: programs declare variables:

$$
\begin{array}{lcll}
P & ::= & D; S & \text{program} \\
D & ::= & var\ x : \tau \mid D; D & \text{Variable declaration}
\end{array}
$$

Suppose the operational semantics discard the variable declaration for now but Γ is defined according to declarations: for a given program $D; S$: $\Gamma(x) = \tau \iff var\ x : \tau \in D$

Problem: how to define Γ in mini-while? (2/2)

Γ can be defined as a typing rule for programs.

From declarations we infer $\Gamma : Var \to Basetype$ with the two following rules:

$$\frac{}{var\ x : t \to_d [x \mapsto t]}$$

$$\frac{D_1 \to_d \Gamma_1 \quad D_2 \to_d \Gamma_2 \quad Dom(\Gamma_1) \cap Dom(\Gamma_2) = \emptyset}{D1; D_2 \to_d \Gamma_1 \cup \Gamma_2}$$

Typing of programs can be defined as follows:

$$\frac{D \to_d \Gamma \quad \Gamma \vdash S}{\emptyset \vdash D; S}$$

Typing a "runtime configuration"

In mini-while, initial state of big step and almost all states of SOS are not just statements, they are pairs: (statement, store). To reason on types at runtime we first need to type runtime configurations:

Definition (Configuration typing)

$$\Gamma \vdash (S, \sigma) \iff (\Gamma \vdash S \land \forall x. \emptyset \vdash \sigma(x) : \tau \iff \Gamma(x) = \tau)$$

Notes:

- $\sigma(x)$ is a value: no Γ is needed to type it
- last part somehow says Γ and σ agree on the type of variables.

Example continued

What is the full program corresponding to the previous example?

```
x = 3 ;
x = x+9 ;
```

What is the initial σ for this program? Does it agree with Γ?

see later ...

2 Imperative languages (C, Mini-While)

- Simple Type Checking for mini-while
- More advanced typing features

Hybrid expressions

What if we have `1.2 + 42` ?

- reject?
- compute a float!

▶ This is **type coercion**. We will see how to implement it during a lab.

▶ It requires a very local form of type inference.

More complex expressions

What if we have types `pointer of bool` or `array of int`? We might want to check equivalence (for addition . . .).

- This is called **structural equivalence** (see Dragon Book, "type equivalence"). This is solved by a basic graph traversal checking that each element are equivalent/compatible.

Sub-typing

- A type can be more precise than another one, e.g.

$$int <: num$$

- Need additional rule to use sub-typing:

$$\frac{e : \tau \qquad \tau <: \tau'}{e : \tau'}$$

- Sometimes, rule to compose sub-types, e.g. functions or parametric types

$$\frac{e : List[\tau] \qquad \tau <: \tau'}{e : List[\tau']}$$

Note: subtyping is heavily used in OOP

3 Type Safety

- Type safety for expressions
- Recall the typing system for mini-while
- Safety

One-step type safety

Theorem (Type correctness for big-step semantics)

If $\emptyset \vdash e : \tau$ *and* $e \longrightarrow v$ *then the value* v *is of the right type* $\emptyset \vdash v : \tau$.

Another one-step reduction is expression evaluation.

Suppose for now Γ and σ agree on types and we prove correctness of expression types as follows:

Prove the following theorem (variant of type correctness)

Suppose $\forall x \in vars(e).\, \emptyset \vdash \sigma(x) : \tau \iff \Gamma(x) = \tau$

prove type correctness for $Val(e, \sigma)$, *i.e.:*

$\Gamma \vdash e : \tau \implies \emptyset \vdash Val(e, \sigma) : \tau$

3 Type Safety

- Type safety for expressions
- Recall the typing system for mini-while
- Safety

Typing recap 1/2

$$\begin{array}{lcll} P & ::= & D; S & \text{program} \\ D & ::= & var\ x : t & \text{type declaration} \end{array}$$

From declarations we infer $\Gamma : Var \rightarrow Basetype$ with the two following rules:

$$\frac{}{var\ x : t \rightarrow_d [x \mapsto t]}$$

$$\frac{D_1 \rightarrow_d \Gamma_1 \quad D_2 \rightarrow_d \Gamma_2 \quad Dom(\Gamma_1) \cap Dom(\Gamma_2) = \emptyset}{D1; D_2 \rightarrow_d \Gamma_1 \cup \Gamma_2}$$

Typing recap 2/2

Then a typing judgment for expressions is $\Gamma \vdash e : \tau \in Basetype$.
Statements have no type and judgement is: $\Gamma \vdash S$.

$$\frac{D \rightarrow_d \Gamma \quad \Gamma \vdash S}{\emptyset \vdash D; S} \qquad \frac{\Gamma \vdash e_1 : \texttt{int} \quad \Gamma \vdash e_2 : \texttt{int}}{\Gamma \vdash e_1 + e_2 : \texttt{int}} \qquad \Gamma \vdash x : \Gamma(x)$$

$$\frac{c \in \mathbf{Z}}{c : int} \qquad \frac{b \in \mathbb{B}}{c : bool} \qquad \frac{\Gamma \vdash S_1 \quad \Gamma \vdash S_2}{\Gamma \vdash S_1; S_2} \qquad \frac{\Gamma \vdash x : \tau \quad \Gamma \vdash e : \tau}{\Gamma \vdash x := e}$$

$$\frac{\Gamma \vdash e : \texttt{bool} \quad \Gamma \vdash S_1 \quad \Gamma \vdash S_2}{\Gamma \vdash \texttt{if } e \texttt{ then } S_1 \texttt{ else } S_2} \qquad \frac{\Gamma \vdash e : \texttt{bool} \quad \Gamma \vdash S}{\Gamma \vdash \texttt{while } e \texttt{ do } S \texttt{ done}}$$

Typing configurations:

$$\Gamma \vdash (S, \sigma) \iff (\Gamma \vdash S \wedge \forall x. \emptyset \vdash \sigma(x) : \tau \iff \Gamma(x) = \tau)$$

Before getting into technical proof

What “wrong behaviours” are prevented by our type-system?

▷ We will try to prove this.

What wrong behaviours are still here?

3 Type Safety

- Type safety for expressions
- Recall the typing system for mini-while
- Safety

Safety = well typed programs do not go wrong

In case of a small-step semantics the proof that "well typed programs do not go wrong" relies on two lemmas:

Well-type programs run without error

Lemma (progression for mini-while)

If $\Gamma \vdash (S, \sigma)$, *then there exists* S', σ' *such that* $(S, \sigma) \Rightarrow (S', \sigma')$
OR there exists σ' *such that* $(S, \sigma) \Rightarrow \sigma'$.

Note: (S, σ) cannot be a final configuration.

... and remain well-typed

Lemma (preservation)

If $\Gamma \vdash (S, \sigma)$ *and* $(S, \sigma) \Rightarrow (S', \sigma')$ *then* $\Gamma \vdash (S', \sigma')$.

Note that Γ never changes (defined by declarations)

Proofs! (recall the property for expression evaluation)

Initial Configuration

Problem: how do we start execution?

Until now $(S, \emptyset)$ was a good starting configuration.

But Γ and $\emptyset$ do not agree on the typing

what is a good initial σ?

Two basic solutions:

- Initialisation or
- consider that $\sigma(x)$ agrees with anything if $x \notin Dom(\sigma)$

Discussion + refer to practical session.

Principle

- Gamma is constructed with lexing information or parsing (variable declaration with types).
- Rules are semantic actions. The semantic actions are responsible for the evaluation order, as well as typing errors.

What is a good output for a type-checker?

- We do not want:

  ```
  failwith "typing error"
  ```

  the origin of the problem should be clearly stated.
- We keep the types for next phases.

In practice

- Input: Trees are decorated by source code lines (and columns).
- Output: Trees are decorated by types in addition.

Type Checking V1 : Visitor

MuTypingVisitor.py

```
# now visit expr

def visitAtomExpr(self, ctx):
    return self.visit(ctx.atom())

def visitOrExpr(self, ctx):
    lvaltype = self.visit(ctx.expr(0))
    rvaltype = self.visit(ctx.expr(1))
    if (BaseType.Boolean == lvaltype) and (BaseType.Boolean == rvaltype):
        return BaseType.Boolean
    else:
        self._raise(ctx, 'boolean operands', lvaltype, rvaltype)
```

In practice for mini-C (lab sessions)

No annotation is added to the AST (everything is int or bool, no ambiguity)

We can create associating type to variables, directly from parsing

Conclusion 1/2

We have seen:

- The properties and principle of static typing
- A type system for miniml
- A type system for mini-while
- Type safety and how to prove it — miniml and mini while
- discussion on variable declaration and initialisation

Conclusion 2/2

Further discussions not really covered in the course:

- Typing functions (later in the course)
- More complex (i.e. real life) type system: sub-typing, objects, functions and sub-typing
- There exist very rich type systems , e.g. behavioural types, linear types, ownership types, ...
  See research on type system and a few modern programming languages.